Bandwidth and dynamic range of a widely tunable Josephson parametric amplifier

M. A. Castellanos-Beltran, K. D. Irwin, L. R. Vale, G. C. Hilton, and K. W. Lehnert

***Abstract*—We characterize the signal bandwidth and dynamic range of a recently developed type of Josephson parametric amplifier. These amplifiers consist of a series array of SQUIDs embedded in a microwave cavity. They are narrow band, only amplifying signals close to the cavity's resonance frequency, but the cavity's resonance frequency, and hence the amplified band, can be widely tuned. For a particular realization of these amplifiers we measure how the signal bandwidth depends on amplifier's gain. We find that the amplitude gain times signal bandwidth is approximately the linewidth of the cavity. In addition we measure the amplifier's dynamic range and saturation power.**

***Index Terms*—Josephson amplifiers, Josephson arrays, parametric amplifiers, gain measurement, SQUIDs.**

I. Introduction

THE ability to manipulate quantum information encoded in microwave fields has led to a renewed interest in Josephson parametric amplifiers (JPAs) [1], [2], [3]. For these applications the ability of JPAs to amplify signals with the least amount of added noise is critical [4], [5], [6], [7]. Unfortunately JPAs are typically narrow band amplifiers with small dynamic range. It is therefore important to understand the bandwidth and dynamic range of any particular JPA in order to determine if it is appropriate for these applications.

We recently introduced a new kind of JPA. Although it is still narrow band, the amplified band can be tuned over a full octave [1]. We have shown that it has good noise performance and can squeeze the vacuum noise by 10 dB [8]. Here we characterize other important parameters of this amplifier, specifically the signal-bandwidth, dynamic range and saturation power.

II. Josephson metamaterial parametric amplifier

Our realization of a JPA consists of a length of nonlinear transmission line from which a half-wavelength cavity is made. The nonlinear transmission line is a coplanar waveguide transmission line where the center conductor is a series array of SQUIDs[1]. The inductance per unit length of this transmission line comes mostly from the nonlinear Josephson inductance of the SQUIDs and not from the geometrical inductance. Furthermore, the inductance per length is tunable with magnetic flux applied to the SQUIDs loops. As a consequence, the phase velocity of waves in this transmission line depends both on intensity of those waves and on the applied flux. At microwave frequencies the SQUIDs are spaced much less than a wavelength apart, so we treat the array as a continuous medium with flux tunable and intensity dependent phase velocity (c.f. an optical Kerr medium). We call this nonlinear effective medium a Josephson metamaterial (Fig. 1).

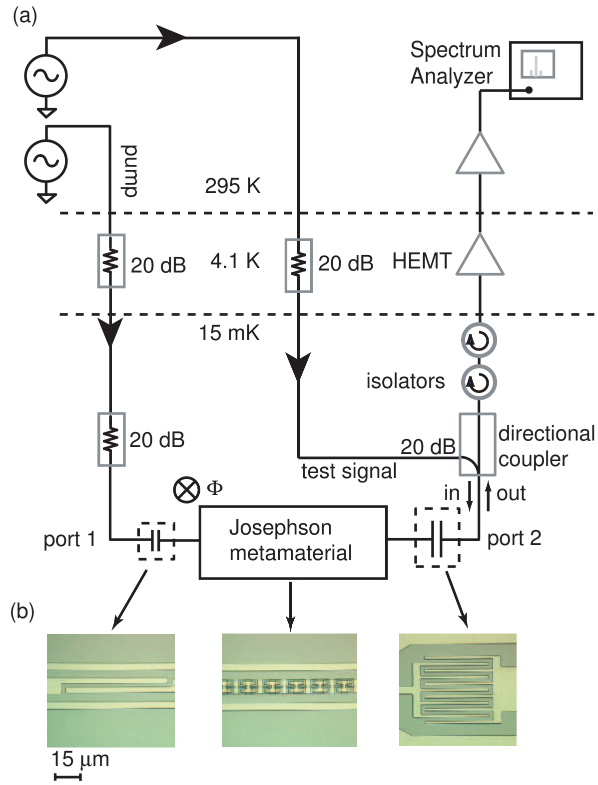

Fig. 1. (a) Measurement schematic. The device is cooled down to 15 mK using a dilution cryostat. Two microwave generators are used to study the JPA: one creates the pump tone, while the second creates a signal tone. The pump is injected through the weakly coupled port (port 1), while the signal tone is incident on the strongly coupled port (port 2). Signals emerging from port 2 go through a pair of isolators before being amplified by a cryogenic high-electron-mobility transistor amplifier (HEMT) and a room temperature amplifier. (b) Pictures of the device.

We create a half-wavelength microwave cavity from this metamaterial by interrupting the center conductor with two capacitors, one much bigger than the other, thus creating a strongly coupled port and a weakly coupled port (Fig. 1). Because the resonance frequency (f_{res}) of this cavity is proportional to the phase velocity, it is dependant on the intensity of the microwave field stored in the cavity. When a large tone, referred to as the pump, is applied through the

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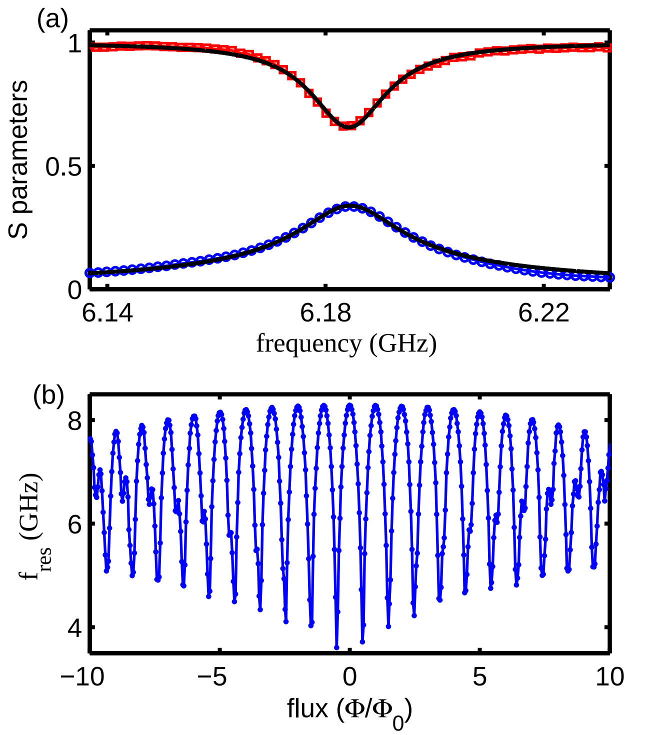

Fig. 2. (a) Magnitude of the transmission $|S_{21}|$ (circles) and reflection $|S_{22}|$ (squares) coefficients as a function of frequency at $\Phi = 0.37\Phi_0$. From the fits (lines), we extract the resonance frequency $f_{res} = 6.184$ GHz, and the amplitude decay rates associated with port 1, port 2 and internal loss $\gamma_{c1} = 2\pi \times 323$ kHz, $\gamma_{c2} = 2\pi \times 7.805$ MHz, and $\gamma_i = 2\pi \times 1.277$ MHz, respectively. (b) Resonance frequency as a function of magnetic flux (points connected by lines)

weakly coupled port close to the resonance frequency of the cavity, it makes the resonance frequency oscillate at twice the pump frequency. This process creates parametric gain[9].

III. Design and Fabrication

Devices were fabricated at NIST Boulder on a high purity uncompensated silicon wafer with resistivity > 17 kΩcm. A standard Nb/AlOx/Nb trilayer process [10] was used, modified by eliminating the shunt resistor layer and minimizing deposited oxides [11]. Some of the features of this sample were already explained in a previous letter [8], so we will only mention the measured parameters. The SQUIDs have an average critical current I_c per SQUID in the JPA of 31 μA. This value is close to the designed value of 30 μA. The coplanar waveguide transmission line was designed to have a capacitance per unit length of $C_l = 0.15$ nF/m and a geometrical inductance per unit length of $L_l = 0.49$ μH/m. The coupling capacitances were estimated to be 4.6 fF and 22.6 fF using microwave simulations. Based on measurements of the S-parameters for low powers (Fig. 2a), these values are very close to the observed ones. From the expected impedance and phase velocity of the transmission line as well as the coupling capacitors' values, we predicted a half-wave resonance frequency of 8.23 GHz with no applied flux. This value agrees with the measured resonance frequency within 3% (Fig. 2b).

In Fig. 2b we show how the flux tunes the cavity's resonance frequency over many periods. Such a plot provides a qualitative measure of the uniformity of the SQUIDs that comprise the device. In particular, the uniformity is much better than the device in [1] which was fabricated using electron beam lithography and double angle evaporation.

IV. Operation

The JPA is operated in four photon mode: as described in section II, the pump tone, with frequency f_p, is applied through the weakly coupled port close to the resonance frequency of the cavity. The large pump makes the resonance frequency oscillate at twice the pump frequency, creating parametric gain [9]. The JPA operates as a reflection amplifier as shown in Fig. 1(b). When the signal with frequency f_s is detuned from the pump, the parametric amplifier will amplify that signal, and create another amplified tone at a frequency of $f_i = 2f_p - f_s$, usually referred to as the idler or intermodulation tone. As we increase the pump in order to obtain higher gains, the optimal pump frequency will shift down as a consequence of the Kerr-nonlinearity of the cavity.

V. Results and Analysis

We study the amplifier's properties using the apparatus shown in Fig. 1a. As described in [1], we determine the critical pump power at which the JPA gain diverges. We then characterize the intermodulation gain G_I and direct gain G_D for pump powers less than this critical power. We define G_D as the ratio between the incident and reflected signal power; G_I is the ratio between the intermodulation tone and the incident signal power. To characterize the bandwidth of the parametric amplifier, we measure the frequency dependance of both gains. For sufficiently large gains, these are Lorentzian functions of the detuning δf between the signal and pump frequencies. For each pump power we determine the 3 dB bandwidth $\Delta f_{1/2}$ and gain at $\delta f = 0$. We observe that with increasing gain, $\Delta f_{1/2}$ is reduced. We find that the expression $\sqrt{G_D}\Delta f_{1/2} \approx \gamma$ is correct, within a factor of two [12], where γ is the half width at half-maximum of the resonator, and is given by $\gamma = \gamma_{c1} + \gamma_{c2} + \gamma_i$. This expression is also correct for G_I if $G_I > 4$. These results are shown in figs. 3(a) and 3(b), where we plot both G_D and G_I as a function δf for three different pump powers for $f_{res} = 6.184$ GHz ($\Phi = 0.37\Phi_0$). Qualitatively, this behavior is expected for parametric amplifiers, and we find quantitative agreement with the theory of [12]. From this plot, we estimate the 3 dB bandwidth to be about 1.38 MHz when G_D and G_I are 22 dB. Although we can observe some internal loss, this loss won't significantly affect the gain or the bandwidth of the amplifier as long as $\gamma_i < \gamma_{c1} + \gamma_{c2}$. However, any loss will lead to some additional noise added by the amplifier, a quantity which we recently measured [8].

For low enough powers in the signal, the gain of the amplifier will be linear, i.e., the output power of the amplifier will depend linearly with the input power. However, as we increase the signal power we will start saturating the JPA, as shown in Fig. 4a. We have estimated saturation power and the dynamic

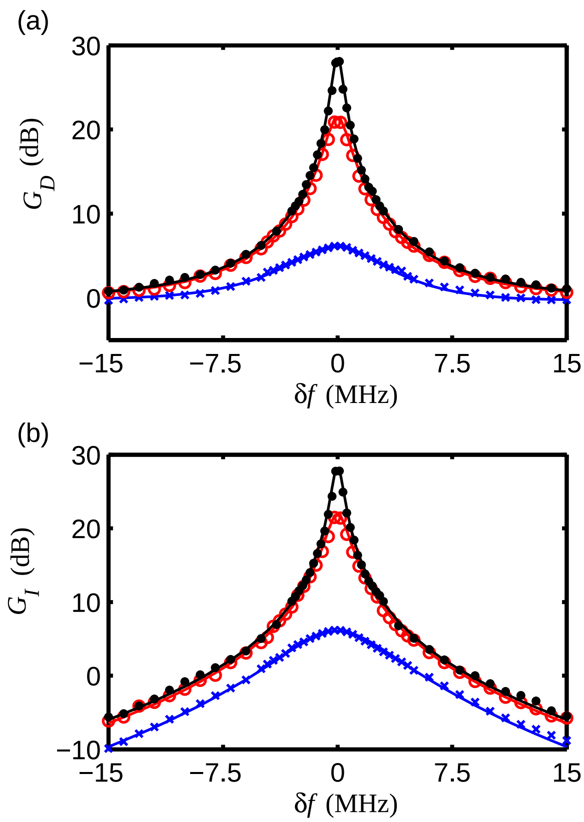

Fig. 3. Gain measurements. (a) and (b) Direct and and intermodulation gain as functions of signal-pump detuning (points) and predictions of [12] (lines) for three different pump powers and frequencies: $P = 0.95\ P_c$ and $f_p = 6.176$ GHz (dots), $P = 0.9\ P_c$ and $f_p = 6.1768$ GHz (circles), and for $P = 0.6\ P_c$ and $f_p = 6.1806$ GHz (crosses). For each pump power, we find an optimum pump frequency. The optimum frequency decreases with increasing pump power due to the Duffing-like behavior of the resonator [12]. Then we sweep the signal frequency over a range around the optimum pump frequency. The signal power used to measure the gains was -160 dBm.

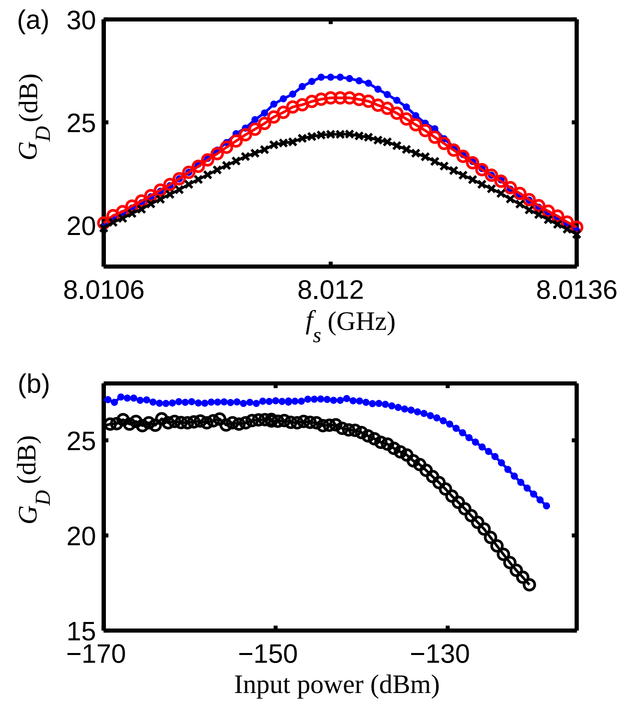

Fig. 4. (a) Direct gain as a function of signal frequency for $\Phi = 0.15\Phi_0$ for three different signal powers, and $f_p = 8.0121$ GHz, showing the expected saturation behavior at large enough signal powers. Signal powers shown are -145 dBm (dots), -130 dBm (circles) and -125 dBm (crosses). (b) Direct gain as a function of input signal powers for $f_p = 6.1766$ GHz (circles) and $f_p = 8.0121$ GHz (dots). The signals were applied 60 kHz above the pump frequency.

range of this amplifier at 2 different resonant frequencies. We determine the saturation power as the input power for which the gain is reduced by 1 dB (Fig. 4b). In Fig. 4b we plot G_D as function of input signal power for the two cavity resonance frequencies. At 8.012 GHz with a pump power of $P = -76$ dBm and $G_D = 27$ dB, the 1 dB compression point is -130 dBm. From noise measurements described in [8], the minimum detectable signal for this amplifier in 1 Hz band is $0.77\hbar\omega$. From these two measurements, we estimate the dynamic range to be 74 dB/Hz. At 6.177 GHz with a pump power of P= -84 dBm and $G_D = 26$ dB, the 1 dB compression point is -137 dBm and the dynamic range is 67 dB/Hz. The reduction in dynamic range is expected: when we tune down the resonance frequency by applying a magnetic field, we are reducing the critical currents of the SQUIDs and the critical power of the resonator.

As a rule of thumb, we have observed that the saturation powers for large gains can be found as $P_{sat} \approx (\gamma_{c1}/\gamma_{c2})P_c/(20G_D)$, where P_c is the critical pump power. The factor γ_{c1}/γ_{c2} arises because we applied the pump to the weakly coupled port, but applied the signal to the strongly coupled port. This expression is reasonable as we expect to observe saturation behavior when the amplified signal power becomes a significant fraction of the pump power inside the cavity.

VI. Conclusion

Applications for an amplifier of this type would include amplifying microwave signals that encode the motion of a nanomechanical oscillator or a superconducting qubit. It is clear that for largest bandwidth one would like to operate with the minimum gain necessary. We can make an estimate of the minimum gain by asking what JPA gain would make the noise added by the following amplifier equal to the vacuum noise. If the following amplifier is a state-of-the-art HEMT amplifier operating at 5 GHz and assuming a noise temperature of 5 K, we would require 16 dB of gain. For this realization of a JPA, the bandwidth would then be approximately 2 MHz. This bandwidth is well suited for detecting nanomechanical oscillators of the type in [13], but marginal for measuring the state of a superconducting qubit [14]. However, it is plausible to increase the gain-bandwidth product of the JPA by implementing larger coupling capacitors, hence increasing the cavity linewidth.

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